Revival of Single-Particle Transport Theory for the Normal State of High-T_c Superconductors: II. Vertex Correction

O. Narikiyo [*]

(Jan. 25, 2013)

Abstract

The vertex correction for the electric current is discussed on the basis of the Fermi-liquid theory. It does not alter the qualitative description of the electric transport by the relaxation-time approximation in the case of the normal state of high-T_c superconductors. The failure of the transport theory employing the fluctuation-exchange (FLEX) approximation is pointed out. It becomes manifest by considering the Ward identity for the fluctuation mode.

In the previous Short Note, arXiv:1204.5300v3, the DC Hall conductivity is discussed on the basis of the relaxation-time approximation. Such an approximation is enough to explain the qualitative features of the measured DC Hall conductivity in the normal state of high-T_c superconductors. The vertex correction is irrelevant. The reason is discussed in this Short Note. Especially the failure[1] of the transport theory based on the fluctuation-exchange (FLEX) approximation is pointed out. In the next Short Note I shall discuss the AC Hall conductivity.

The discussion on the vertex correction for the electric current based on the Fermi-liquid theory is reviewed in Chap. VIII of [1]. More explicit diagrammatic representation is given in [2]. The following discussion is based on these [1, 2].

---

[*]Department of Physics, Kyushu University, Fukuoka 812-8581, Japan
[1] I have enforced the critiques reported in cond-mat/0006028v1, 0012505v1, 0103436v1.

We start from the self-energy for electrons shown in Fig 1 where the electrons with $\mathbf{k}$ and $\mathbf{k}{-}\mathbf{q}$ carry spin σ and the others with $\mathbf{k}'$ and $\mathbf{k}'{+}\mathbf{q}$ carry spin σ'. The diagrammatic representation for the electric current vertex $\Lambda_\mu(\mathbf{k})$, where $\mu = x, y, z$, is also given in Fig. 1. The vertex correction consistent with the Ward identity is obtained as Fig. 2 by introducing the coupling to external field to one of the internal lines of the self-energy. Thus the integral equation to determine $\Lambda_\mu(\mathbf{k})$ is

$$\Lambda_\mu(\mathbf{k}) = J_\mu(\mathbf{k}) + \Lambda_\mu^{(a)}(\mathbf{k}) + \Lambda_\mu^{(b)}(\mathbf{k}) + \Lambda_\mu^{(c)}(\mathbf{k}), \qquad (1)$$

where $J_\mu(\mathbf{k})$ is the bare vertex, $\Lambda_\mu^{(a)}(\mathbf{k})$ is the integral depicted in Fig. 2-(a) and so on. This equation is rewritten as[2]

$$0 = J_\mu(\mathbf{k}) + \sum_{\mathbf{k}'} \sum_{\mathbf{q}} \Delta_0(\mathbf{k}, \mathbf{k}'; \mathbf{k}'{+}\mathbf{q}, \mathbf{k}{-}\mathbf{q})$$
$$\times \left[\Phi_\mu(\mathbf{k}{-}\mathbf{q}) + \Phi_\mu(\mathbf{k}'{+}\mathbf{q}) - \Phi_\mu(\mathbf{k}') - \Phi_\mu(\mathbf{k}) \right], \qquad (2)$$

at low temperatures where the Fermi degeneracy becomes a strong constraint. This equation is consistent with the collision term[3] in the Boltzmann equation. Such a form is the consequence of the Fermi statistics (the Pauli principle).

On the other hand, in the FLEX approximation [3] the Aslamazov-Larkin processes[4] corresponding to $\Lambda_\mu^{(b)}(\mathbf{k})$ and $\Lambda_\mu^{(c)}(\mathbf{k})$ vanish and only the Maki-Thompson process corresponding to $\Lambda_\mu^{(a)}(\mathbf{k})$ contributes to the integral equation. Thus this approximation does not lead to the vertex obeying Eq. (2) and is not applicable to the system with the Fermi degeneracy.

The reason why the Aslamazov-Larkin processes vanish has nothing to do with the Umklapp scattering[5]. They vanish for any charge-neutral fluctuation mode. It is rigorously shown by the Ward identity[6].

A correct transport theory for electrons at low temperatures[7] should obey the Pauli principle. However, the replacement of the renormalized interaction (depicted by the square) by the fluctuation mode in the FLEX approximation breaks the Pauli principle [4, 5, 6]. Only internal consistency is guaranteed

---

[2] See Eqs. (6·13) and (6·17) in [2].

[3] See Eqs. (40.2) and (40.6) in [1].

[4] See Fig. 13 in [3].

[5] The role of the Umklapp scattering is stressed in [3].

[6] See arXiv:1212.6484v1 and references therein.

[7] At low temperatures the transport theory should based on quasi-particles obeying the Pauli principle. On the other hand, the superconducting fluctuation transport theory is formulated in high-temperature limit.

by the FLEX approximation but the difference between the exact result and the approximation cannot be estimated in its framework.

As discussed in §39 of [1] the vertex correction for elastic scatterings removes the contribution of the forward scattering[8]. This effect is irrelevant[9] to the qualitative discussion in the case of high-T_c superconductors where the forward scattering is negligible.

Consequently the relaxation-time approximation is enough to describe the qualitative features of the transport in the normal state of high-T_c superconductors.

In the previous Short Note, arXiv:1204.5300v3, it is clarified that the qualitatively correct self-energy is necessary to obtain the transport coefficients consistent with experiments. However, such a self-energy cannot be obtained [6] by the FLEX approximation.

References

[1] A. A. Abrikosov, L. P. Gor'kov, I. Ye. Dzyaloshinskii: *Quantum Field Theoretical Methods in Statistical Physics*, second edition (Pergamon Press, Oxford, 1965).

[2] K. Yamada, K. Yosida: Prog. Thoer. Phys. **76**, 621 (1986).

[3] H. Kontani: Rep. Prog. Phys. **71**, 026501 (2008) and references therein.

[4] N. E. Bickers, S. R. White: Phys. Rev. B **43**, 8044 (1991).

[5] J. J. Deisz, D. W. Hess, J. W. Serene: Phys. Rev. B **55**, 2089 (1997).

[6] Y. M. Vilk, A. M. S. Tremblay: J. Phys. I (France) **7**, 1309 (1997).

---

[8] It is represented by the factor $(1 - \cos\theta)$ in Eq. (39.17) of [1].

[9] See, for example, B. P. Stojkvić and D. Pines: Phys. Rev. B **55**, 8576 (1997).
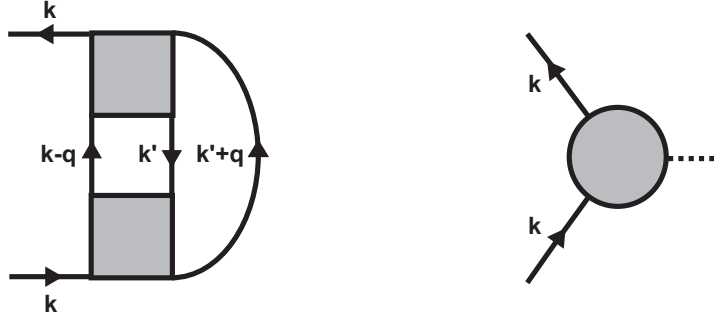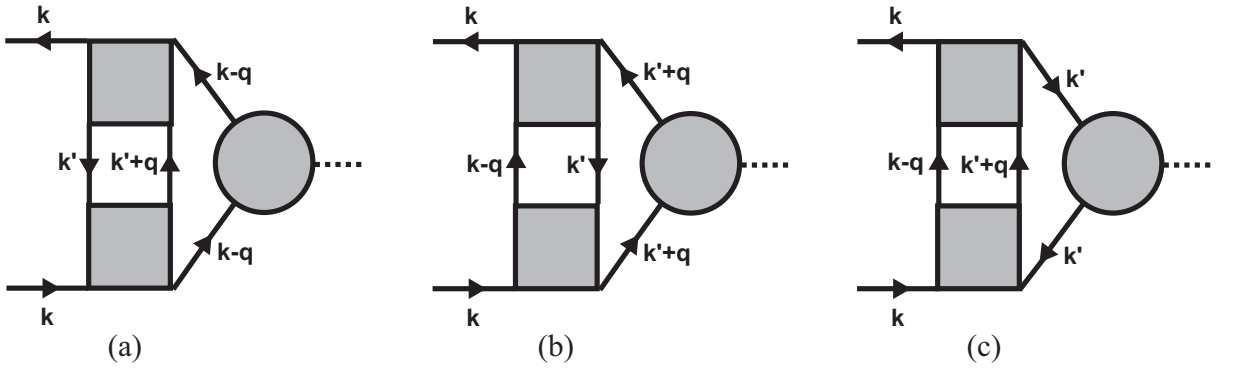
Figure 1: (Left) Self-energy. (Right) Current vertex.

Figure 2: Vertex corrections.